

Number and data representation

Binary, hexadecimal, and more

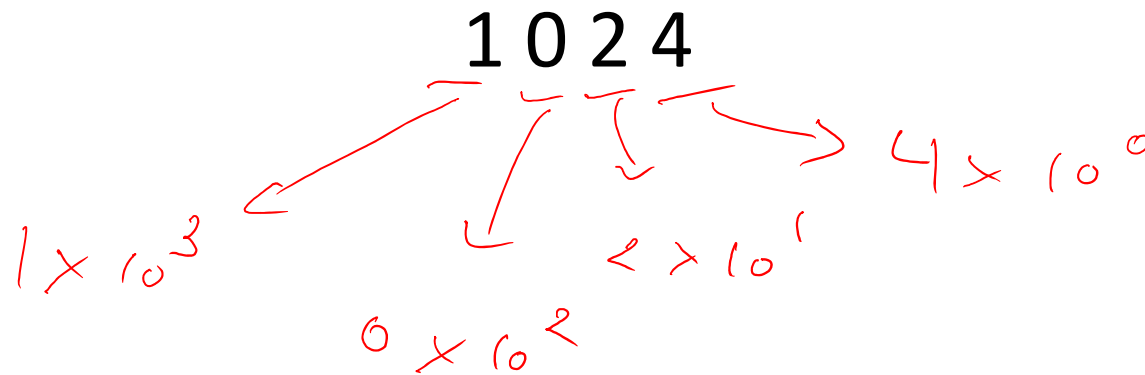
Numbers and counting

- Why do we count in base 10?



Big idea: Positional encoding

Base 10:



Base 8:

OCTAL

$$34_8 = 3 \times 8^1 + 4 \times 8^0$$

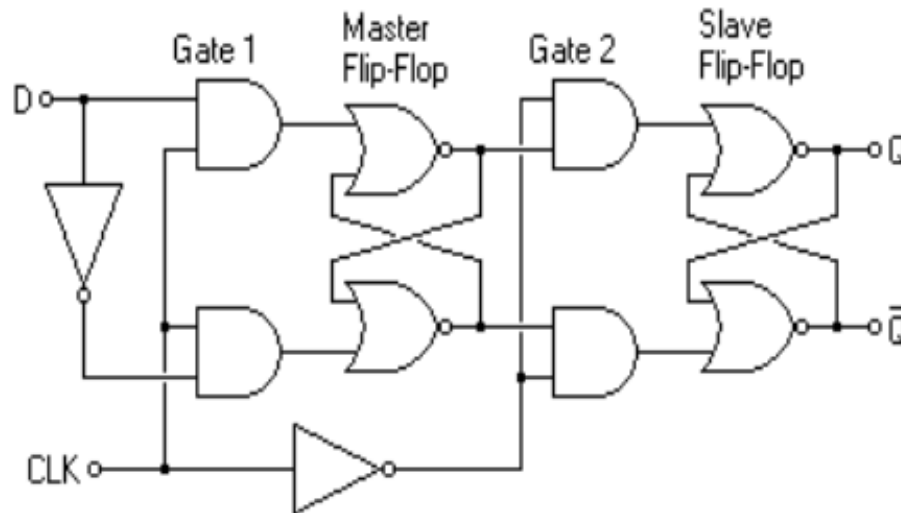
$$a \times b^e$$



Yoda says: "Positional encoding is great for doing computation, the base is not that big of a deal ... unless you are working on a computer"

Binary representation (base 2)

- Only two symbols: 0 and 1
- Internally represented as voltages: LOW and HIGH
- Why only 2?



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Binary representation (base 2)

- Only two symbols: 0 and 1
- Each position is called a *bit*
- What is the decimal equivalent of the binary number:

$$\begin{array}{ccccccc} 0 & 1 & 0 & 1 & & = & 5 \\ \leftarrow 2^3 & \leftarrow 2^2 & \leftarrow 2^1 & \leftarrow 2^0 & & & \\ 32 & 16 & 8 & 4 & 2 & 1 & \end{array}$$

Decimal to binary

$$45_{10} = ?$$

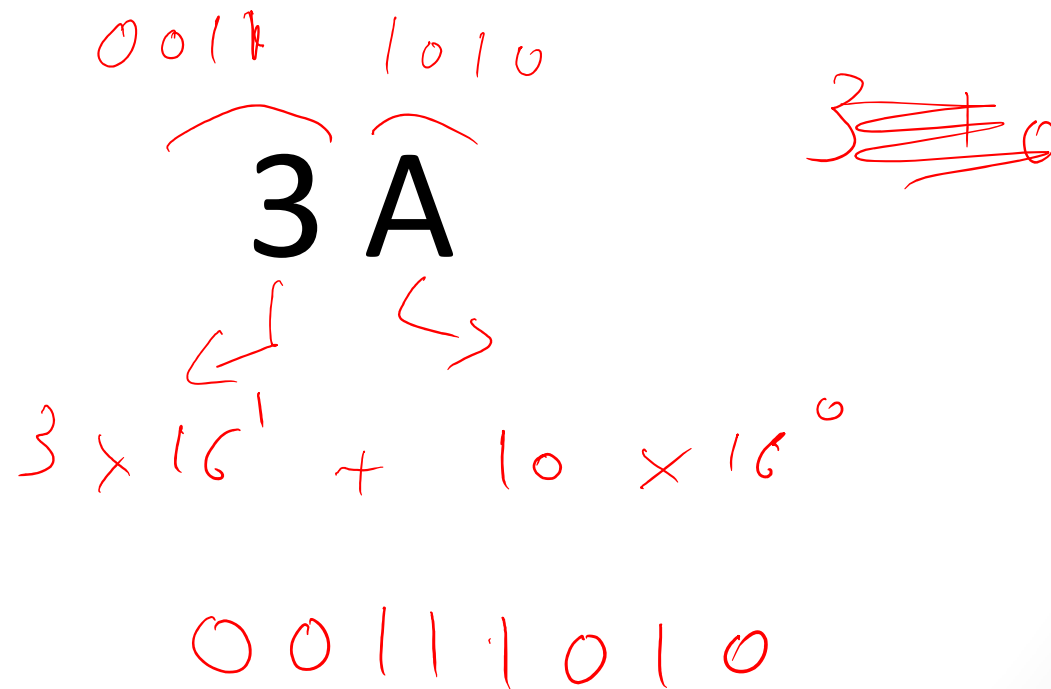
Represented as an 8 bit binary number, has how many bits that are 1?

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

Hexadecimal (base 16)

- 16 possible symbols:

0 1 2 3 4 5 6 7 8 9 A B C D E F



Binary?

Hexadecimal (base 16)

DEC *HEX*

0	0	8	8
1	1	9	9
2	2	10	A
3	3	11	B
4	4	12	C
5	5	13	D
6	6	14	E
7	7	15	F

Hexadecimal (base 16)

DEC	HEX	BIN	DEC	HEX	BIN
0	0	0000	8	8	1000
1	1	0001	9	9	1001
2	2	0010	10	A	1010
3	3	0011	11	B	1011
4	4	0100	12	C	1100
5	5	0101	13	D	1101
6	6	0110	14	E	1110
7	7	0111	15	F	1111
			16	10	10000
			17	11	10001

Conversions

Binary, hexadecimal, decimal, octal, ...

01101110

Handwritten annotations in red ink:

- A red circle around the first '0'.
- Vertical lines separating the binary string into groups: 0, 11, 01, 11, 10.
- Hexadecimal digits written below the groups: 1, 6, 5, E, 6.
- A red '4' written above the '01' group.

Representing other data

- All the information in your computer is in binary –how?
- Big idea: N bits can be used to represent 2^N things!

Binary patterns

0001

0110

10

Numbers




Colors

GREEN

RED

Bits take up space

- How many bits would you need (at the minimum) to represent 10 different colors?

- A. 2  4
- B. 3  8
- C. 4  16
- D. 5
- E. 10

Binary operations




- Addition







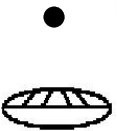
$$\begin{array}{r} 1011 \\ + 0111 \\ \hline 1000 \end{array}$$

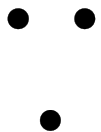
- Shift

$$\begin{array}{r} 0011 \quad 3 \\ 00110 \quad 6 \\ 001100 \quad 12 \end{array}$$









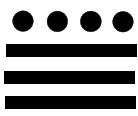
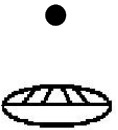
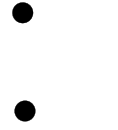
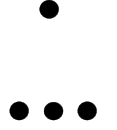
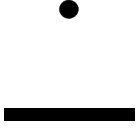


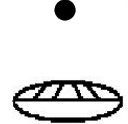
Maya Numbering System

							
1	2	5	6	9	10	13	15

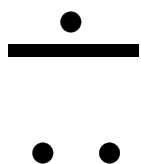
							
19	20	21	23	25	40	100	400



Maya Numbering System

							
1	2	5	6	9	10	13	15
							
19	20	21	23	25	40	100	400

- What decimal number does this Maya symbol represent?



- | | |
|-------|--------|
| A. 26 | D. 122 |
| B. 46 | E. 242 |
| C. 62 | |